Bandwidth scaling and spectral flatness enhancement of optical frequency combs from phase-modulated continuous-wave lasers using cascaded four-wave mixing

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We introduce a new cascaded four-wave mixing technique that scales up the bandwidth of frequency combs generated by phase modulation of a continuous-wave (CW) laser while simultaneously enhancing the spectral flatness. As a result, we demonstrate a 10 GHz frequency comb with over 100 lines in a 10 dB bandwidth in which a record 75 lines are within a flatness of 1 dB. The cascaded four-wave mixing process increases the bandwidth of the initial comb generated by the modulation of a CW laser by a factor of five. The broadband comb has approximately quadratic spectral phase, which is compensated upon propagation in single-mode fiber, resulting in a 10 GHz train of 940 fs pulses. © 2012 Optical Society of America

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Strong sinusoidal phase modulation of a continuous wave (CW) laser creates multiple sidebands leading to generation of a frequency comb \cite{1}. The advantages of this technique are the ability to create high repetition rate combs with stable optical center frequencies given by the source laser and convenient tuning of the repetition rate and optical center frequency. Such combs are a source of choice for applications in optical communications \cite{2}, radio frequency (RF) photonics \cite{3,4} and optical arbitrary waveform generation (OAWG) \cite{5}. The bandwidth of such combs, however, is limited. The number of spectral lines scales linearly with the RF voltage driving the phase modulator. RF power handling limits the number of lines that can be generated by a single phase modulator, requiring a cascade of phase modulators to generate more lines. For example, state-of-the-art commercially available, low Vpi phase modulators (Vpi ∼3 V) usually have a RF power limit of ∼1 W, which limits the number of lines to ∼20 in a 3 dB bandwidth (e.g., 200 GHz bandwidth at 10 GHz drive). To reach the 100 line level, we would have to cascade five modulators, which is both expensive and inefficient.

Furthermore, by phase modulation alone, the spectral flatness is quite poor, having significant line-to-line amplitude variations. A strongly modulated spectrum is undesirable for many applications. For example, for pulse train generation, line-to-line variations translate to reduced pulse quality. The flatness problem has been partially addressed by utilizing a Mach–Zehnder intensity modulator in series with the phase modulator \cite{6}. An explanation for the improvement in spectral flatness provided by the addition of the intensity modulator was discussed in \cite{7}. Figure 1 provides a schematic. The intensity modulator is driven such that it creates a flat-topped pulse [indicated by $a_i(t)$ in the figure]. The action of the sinusoidal phase modulation in the window of the flat-topped pulse may be approximated to lowest order as a quadratic temporal phase. Strong quadratic phase in time performs a time-to-frequency mapping operation \cite{8}, creating a comb with a spectral shape similar to the time-domain pulse, which in this case is flat topped.

However, owing to significant deviations of the sinusoid from a quadratic, the comb still has limited flatness with >5 dB spectral variation between the lines in the central region. Recently, we proposed a technique to significantly flatten the spectrum: the duty factor of the flat-topped pulse was reduced by utilizing two intensity modulators in series while the RF drive waveform to the phase modulator was shaped to better approximate a quadratic \cite{9}. This allowed for significant improvement in spectral flatness to <1 dB. The bandwidth obtained, however, was limited by the number of phase modulators (∼40 lines using two phase modulators). A method that scales bandwidth without increasing the number of modulators and improves flatness is desirable. There have been some methods to scale the bandwidth by first compressing the comb to a short pulse and then broadening the spectrum via nonlinear propagation in dispersion decreasing fiber or highly nonlinear fiber (HNLF) \cite{10,11,12}. However, the spectral flatness of such combs is degraded. Also, owing to subtle interplay between dispersion and nonlinearity, the generated spectrum is not very stable.

In this work, we introduce a simple scheme which can scale the bandwidth of the comb by several times (5, 7, 9).
etc.) in a stable and known fashion while simultaneously enhancing spectral flatness. By using just one intensity modulator and one phase modular, we generate a comb comprising over 100 lines within 10 dB bandwidth, out of which a record 75 lines are within the 1 dB bandwidth. Furthermore, our scheme allows compression of the comb into a bandwidth-limited train of 940 fs pulses via simple propagation in a dispersive fiber.

Fig. 2(a) shows the experimental setup. A CW laser (CW 1) at frequency $f_1$ is driven using an intensity and phase modulator to generate a partially flattened comb as discussed previously. If $a_1(t)$, which is a flat-topped pulse, and $\phi(t)$, which is a sinusoid, are the amplitude and phase modulation, the output after the IM and PM is $a_1(t) \exp(j\phi(t))$. We include a second CW laser at frequency $f_2$, which is not amplitude modulated but that passes through the same phase modulator; its modulation is $\exp(j\phi(t))$. This is followed by a length of SMF whose length is chosen such that it delays the $f_2$ field by half an RF period (i.e., 50 ps for a 10 GHz drive frequency) relative to the $f_1$ field. For the $f_2$ field, this transforms the phase modulation according to $\exp(j\phi(t)) \rightarrow -\exp(-j\phi(t))$. The motivation is to ensure a constructive bandwidth addition in the four-wave-mixing (FWM) terms between the combs at $f_1$ and $f_2$. This is followed by a fiber amplifier followed by a near zero dispersion, low dispersion slope, highly nonlinear fiber (HNLF), and a bandpass filter to select an appropriate frequency band. Assuming a short length of HNLF with near zero dispersion and low loss, the propagation regime is pure self-phase modulation, which creates a cascade of FWM terms. Looking towards the side of $f_1$, we will have new frequency components created at $2f_1 - f_2$, which will go as

$$[a_1(t)^2 \exp(3j\phi(t))][a_1(t) \exp(j\phi(t))] \exp(-j\phi(t)^*) = a_1(t)^2 \exp(5j\phi(t)).$$

We clearly see that the bandwidth has tripled in this case. The next term in the cascade of FWM terms will occur at $3f_1 - 2f_2$, which would be dominated by the term corresponding to mixing between the comb corresponding to the first FWM term, the comb at $f_1$, and the comb at $f_2$, which goes as

$$[a_1(t)^2 \exp(3j\phi(t))][a_1(t) \exp(j\phi(t))] \exp(-j\phi(t)^*)$$

This indicates a bandwidth scaling of five times. Similarly, if we look at the higher order terms, we will have bandwidths scaling at seven times, nine times, and so on. However, the efficiency reduces owing to increasing phase mismatch for the nonlinear process. An interesting aspect is that the amplitude coefficient of the above terms successively rises to higher powers of $a_1(t)$. Since $a_1(t)$ was chosen to be a flat-topped waveform, this raising to higher powers reduces the duty cycle of the time domain waveform by making the transition regions shorter, as depicted in Fig. 1. By more effectively restricting the intensity to the region where the phase modulation is close to quadratic, more ideal time-to-frequency mapping is achieved, resulting in flatter combs. In the frequency domain, this can be viewed as successive convolution of the initial combs arranged such that the phases add constructively. This has the effect of smoothening out the spectra. Fig. 2(b) shows schematically the bandwidth scaling and increasing spectral flatness in our cascaded four-wave-mixing scheme.

In our experiment, the two CW lasers were chosen to be at 1542 nm ($f_1$) and 1532 nm ($f_2$), respectively. The spacing was chosen such that there is no overlap between combs generated by adjacent FWM terms up to the second-order. The initial laser fields at 1532 nm and 1542 nm have an instantaneous linewidth of $\sim$100 kHz and $\sim$10 kHz, respectively. The phase modulator has a Vpi of $\sim$3 V and is driven by an RF power of $\sim$1 W. The RF oscillator has a 10 GHz frequency, and the first SMF spool is $\sim$300 m, creating the required half-period (50 ps) delay. We use a high power optical amplifier with $\sim$1.5 W output power. The HNLF we use has a length of 100 m, $D = 0$ ps/nm/km and $S = 0.02$ ps/nm$^2$/km. Practically, high conversion efficiency in FWM-based optical processing for lightweight communications has been demonstrated in several experiments using Watt-level fiber amplifiers and commercial low dispersion HNLF, for example, [13,14]. Figure 3 shows the simulated output spectrum for our experiment obtained by a numerical simulation of the nonlinear Schrodinger equation, taking into

![Figure 2](image1.png)  
**Fig. 2.** (Color online) (a) Experimental setup: CW, continuous wave laser; IM, intensity modulator; PM, phase modulator; SMF, single-mode fiber; HNLF, highly nonlinear fiber; Amp, fiber amplifier; and BPF, bandpass filter; and (b) bandwidth scaling of the comb and enhanced spectral flattening.

![Figure 3](image2.png)  
**Fig. 3.** (Color online) Simulation showing bandwidth scaled flat comb generation.
account second-order and third-order dispersion in the fiber. The signal at 1542 nm is taken to be at a relative frequency of 0 THz. We can see a clear bandwidth scaling of successive terms of the FWM terms with progressive flattening of the power spectrum.

Figure 4(a) shows the measured comb around 1542 nm with ~22 lines in a 10 dB bandwidth with medium flatness. The comb around 1532 nm is of similar bandwidth but is even less flat since it is generated by pure phase modulation (see inset). Fig. 4(b) shows the comb generated by the first FWM term centered around 1552 nm. We can clearly see the three times bandwidth enhancement as well as the improvement in spectral flatness compared to the initial comb. Figs. 4(c) and 4(d) show the second and third FWM term with 1562 nm in linear and log scale. We get around 20 mW of power in this spectral region. We clearly see further bandwidth enhancement to five times and the significant improvement in spectral flatness. The new comb has >100 lines in a 10 dB bandwidth with a record 75 of them within 1 dB. We have thus generated flat combs in a convenient fashion without the need for temporal shaping of the phase modulator waveforms as done in [9].

The combs generated in this way have temporal phase very close to quadratic. By Fourier transform relations, the transform of a purely quadratic temporal phase waveform is a quadratic spectral phase. This means that the generated comb has a linear chirp, which can easily be compensated using dispersive media, e.g., a length of single-mode fiber. This allows us to create high quality short pulses from the generated combs with a simple compression scheme. Fig. 5(a) shows the spectral phase of the comb generated at the second-order FWM term measured via the half-repetition rate modulation scheme described in [15]. We see that the measured spectral phase fits very well to a quadratic. The pulse is compressed using ~200 m of SMF. The spectral phase is again measured via the scheme of [15] and used to compute the compressed pulse temporal intensity profile,